Asset Pricing Models

Finance is the only place where one can win a Nobel prize by linearizing.

On the surface, the relationship between risk and return seems straightforward: both should move in the same direction. A rational investor should accept a higher risk only if it is rewarded by a higher expected return, and decreasing risk should result in lower expected returns.

In practice, risk is usually approximated to volatility, which is a measure of uncertainty. Yet, risk and uncertainty are not exactly the same concept. In the case of risk, we know all the possible outcomes and the probability of their occurring, but we do not know which outcome will occur for sure. In the case of uncertainty, on the other hand, we are unaware either of the possible outcomes or of the probability of their occurring, or both. In a sense, risk and uncertainty are polar extremes, but many situations in the real world have elements of both: we usually believe that we have some knowledge of both probabilities and outcomes. In this context, quantifying precisely the risk–return relationship is hard – if we knew exactly how and when risk is rewarded, it would not be called risk any more.

Investing without knowing what the risks are and how much reward we can expect for taking them is a daunting task. Fortunately, financial theory includes a large body of research called asset pricing, which focuses uniquely on formalizing the relationship between risk and expected returns. For instance, asset pricing explains why the long-term expected return on a short-term government bond should be smaller than the long-term expected return on a stock, or more generally, why two different assets have different expected returns. In addition, asset pricing can help us understand why expected returns may change with time and how the returns on various assets vary together.

Asset pricing has the particularity of being a self-sufficient field of research. Theorists develop models with testable predictions, and empiricists document pricing puzzles. That is, they identify stylized facts that fail to fit established pricing theories and these facts then stimulate the development of new theories. Thanks to this fertile cycle, over the last 50 years, asset pricing has given birth to some of the greatest success stories of neo-classical economic analysis. Among these, we may cite the Capital Asset Pricing Model (CAPM), the Arbitrage Pricing Theory (APT) and the Black–Scholes (1973) option pricing formula. All these models have provided answers and insights that are now indispensable not only to researchers but also to all practitioners in financial markets. Part of their success is obviously attributable to their ability to identify arbitrage opportunities and generate trading ideas. If a market does not obey a model’s predictions, academics usually decide that the model needs improvement and go back to work. But practitioners are happy to live with the idea that markets are sometimes wrong and do not price correctly some assets, because it means potential trading opportunities for the shrewd investor.

The taxonomy of asset pricing models comprises two major categories, namely, absolute pricing models and relative pricing models. Absolute pricing models are most common in academic settings. They include fundamental equilibrium models and consumption-based
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models, as well as most macroeconomic models. Absolute pricing models price each asset individually, by reference to its exposure to fundamental sources of risk. They use asset pricing theory positively to give an economic explanation for why prices are what they are and why agents have such risk exposures, or in order to predict how prices might change if the policy or economic structure changes. Relative pricing models, in contrast, should be considered as a means of exploring the evidence rather than of arriving at a profound behavioral explanation of financial markets. In a sense, they aim at providing a simple representation of how the world works. They price each asset by reference to the prices of some other assets, which are exogenous—the pricing model does not ask where they come from. The Black–Scholes formula is a classic illustration of such a model. It calculates an option price as a function of its underlying asset price, regardless of whether the latter is fairly valued by the market or not.

Not surprisingly, pure theoreticians of finance dislike relative pricing models. They claim that only absolute pricing is worthy of study. Summers (1985), for example, compares financial economists with “ketchup economists”, obsessed only with the relative prices and interrelationships between different sized bottles of ketchup. He concludes, “They ignore what seems to many to be the more important question of what determines the overall level of asset prices”. Needless to say, most practitioners strongly disagree with that opinion. On a day-to-day basis, they need positive and pragmatic models that capture the way the world works, not normative models that state how the world should be. It is naturally this path that we will explore in this chapter.

Most of the asset pricing models we consider below are relatively simple. All fall into the relative pricing category. They price hedge funds relative to the market or other risk factors, and do not really focus on what determines the underlying factors, the market or factor risk premium, or the risk exposures taken by hedge fund managers. Hence, they are relatively easy to understand and apply in practice, provided enough data is available. Most of them are based on a two-step approach.

First, they assume that hedge fund returns are specific functions of carefully selected macro- and microeconomic factors. The selection of relevant factors and the determination of the relationship that links them to fund returns is the key to this approach. Just because a certain fundamental characteristic aligns with past performance does not mean that this characteristic represents a systematic risk factor that the market compensates with some return. This is precisely where the difference between correlation and asset pricing models lies.

Second, using multiple regression analysis, they assess the sensitivity of hedge fund returns to these factors and attempt to validate or invalidate empirically their initial assumption. It is important to realize at this stage that testing any asset pricing model is in fact a joint test of market efficiency and of the validity of the pricing model. Since we do not know if the asset pricing model in question is valid, the acquisition of returns in excess of what the model says may indicate that the hypothesized model is wrong and not that the market is inefficient.

8.1 WHY DO WE NEED A FACTOR MODEL?

Before going any further, let us explain briefly why factor models are useful, not only for asset pricing, but also for portfolio management, risk measurement and, more generally, for any discipline that needs information about the co-movements of different assets (i.e. uses the covariance and/or the correlation matrix).
8.1.1 The dimension reduction

Technically, an asset pricing factor model does no more than condense the dynamics of a large series of fund returns into a smaller series of explanatory factors, whose influence is common to all funds. In a sense, the small set of factors and factor exposures\(^1\) provides a parsimonious representation of the large set of funds. That is, it explains most of the variance and covariance of the funds considered.

To understand the importance of this dimension reduction, consider the following example. Suppose we have to monitor a universe of \(N = 1000\) hedge funds. We may decide to analyze each individual hedge fund and its particularities rather than trying to build a “one-size-fits-all” factor model. And when building portfolios, we may simply analyze the correlation or covariance between the returns of all funds, so that we can figure out which funds will complement well other funds in terms of diversification. However, monitoring a universe of a thousand funds implies estimating (among other things) a covariance matrix of one million terms \((N \times N)\). Even bearing in mind that the covariance matrix is symmetrical, so that we only need to estimate \(N \times (N - 1)/2\) terms, this still represents 499,500 terms. This is a manifestation of the so-called curse of dimensionality. The time series necessary to estimate so many parameters from historical data with an acceptable degree of measurement error is enormous, as is the computation time. And we have not yet talked about out-of-sample forecasting, which is necessary to build portfolios with a good diversification of risks in the future! Clearly, we have to find another approach.

The element that requires the largest number of estimates is the covariance matrix. It is therefore the crux of the matter. To simplify the analysis of individual funds as well as the process of portfolio construction, we need to simplify the estimation of the covariance matrix. One possible solution is to make some simplifying assumptions about its structure, that is, assumptions about the interrelationships among funds. This is exactly what factor analysis does. Factor analysis determines from a statistical perspective the interrelationships among a large number of variables (e.g., fund returns) and expresses these interrelationships in terms of their common underlying dimensions (the factors). The output of factor analysis is a factor model, where the return of each fund is represented as the sum of two mutually orthogonal components: a component that is common to all funds, and a component that is idiosyncratic to each fund. The common component is driven by a small number of factors that influence all the funds in the model. The idiosyncratic component is driven by fund-specific shocks.

As mentioned already, the advantage of factor analysis is the dimensionality reduction. Factor analysis condenses the information contained in a large number of original variables into a smaller set of factors with minimum loss of information. Say, for instance, that we are able to identify \(K = 5\) factors that explain most of the behavior of our \(N = 1000\) funds. In this case, we just need to estimate how our funds react to our five factors (i.e., 5000 coefficients) as well as what is the covariance structure of our factors (i.e., 10 coefficients). We have reduced the dimensionality of the problem from 499,500 estimates to 5010. Although the latter number is still large, it is much more reasonable than the former.

In practice, the use of factor analysis is supported by the observation that hedge fund returns tend to react together to some extent, particularly if we consider funds following the same type of strategy. This confirms the intuition that fund returns are likely to be affected by the same factors at the same time. Therefore, it is meaningful to attempt to capture the common behavior

\(^1\) In econometric jargon, “factor exposures” are sometimes called “factor loadings”.
of a series of funds by one or several factors. As we will see shortly, factor models lie at the heart of modern portfolio management and risk analysis, both for backward-looking portfolio evaluation and for forward-looking portfolio structuring and rebalancing. In particular:

- A factor model is useful in understanding why a portfolio had a certain return over a particular period of time. It throws light on the risks taken by an investment manager to achieve those returns and provides a decomposition of returns in terms of various bets made by the manager by over- or underweighting the exposure to certain risk factors.
- A factor model is essential for certain investment strategies that need to precisely attribute the risk to different sources. For instance, this is the case for indexing (where one tries to replicate some risk exposure) or market neutral portfolios (where zero exposure to specific sources of risk should be maintained continuously).
- A factor model is necessary to predict return, volatility and correlation figures in a consistent way. In particular, a factor model can capture time-varying features of volatility and achieve more accurate forward-looking risk forecasts.
- A factor model should help identify the value added by the manager over a passive portfolio.

However, to be effective, a factor model has to possess a minimum number of properties. First, it must be feasible to estimate its parameters in a reasonable amount of time. Second, it has to be intuitive to use. Third, it has to be parsimonious enough in terms of number of factors to avoid overfitting and guarantee adequate out-of-sample performance. And finally, it must reflect commonalities in fund returns in order to reduce noise and to achieve the decompositions desired in making investment decisions such as hedging, benchmarking, performance attribution and segmented analysis.

An important point to consider with factor models is whether we need a predictive model, an explanatory model, or both. The distinction is important, because explaining past covariance and forecasting future covariance are completely different activities. Using the same factor model for both predictive and explanatory purposes implicitly assumes that fund returns are influenced by factors that persist over time, and that there is some stability in factor exposures. So far, the vast majority of factor models suggested in the literature have employed ex post (observed) returns as a proxy for expected returns. In theory, this should not be a problem because the two series should converge. However, in practice, the rate of convergence critically depends on the assumption of rational agents’ expectations, which itself is not necessarily guaranteed in environments in which investors have limited amounts of data and attempt to understand the complicated dynamics of the underlying economy. In the following discussion, we concentrate essentially on explanatory factors. We therefore express all our models in ex post form, i.e. we do not consider expectations. We revert later to the question of forecasting and predictability of hedge fund returns.

### 8.2 LINEAR SINGLE-FACTOR MODELS

Since the time of Newton, common scientific expertise has advanced by using linear equations to model most natural phenomena. It is therefore not surprising that the first asset pricing models we are going to consider are linear with respect to their factors. That is, they postulate that rates of return on all funds are related linearly to a set of $K$ factors. This offers the advantage of being able to use linear regression analysis to estimate the parameters of the model.
Return on fund $i$

\[ R_i \]

\[ \beta_i \]

Abnormal return $\alpha_i$

Normal return $F$

Statistical noise $\varepsilon_i$

Figure 8.1  The three sources of return and risk in a single-factor model

\[ R_i = \alpha_i + \beta_i \cdot F + \varepsilon_i \] (8.1)

\[ \varepsilon_1 \]

\[ \varepsilon_2 \]

\[ \varepsilon_N \]

Fund 1

Fund 2

Factor

Fund $N$

Figure 8.2  Fund returns are only related to each other through their common relationship with the factor

8.2.1 Single-factor asset pricing models

The simplest asset pricing model uses only one factor ($K = 1$) and expresses the return on each fund as a linear function of a factor $F$. Thus, fund returns are only related to each other through their common relationship with the factor $F$. In this case, we can write

where $\alpha_i$ and $\beta_i$ are parameters to be estimated, and $\varepsilon_i$ denotes an idiosyncratic error term, which is assumed to be zero on average and uncorrelated with the common factor. Estimates of $\alpha_i$ and $\beta_i$ are usually obtained by regression analysis, and usual statistical significance tests may be used to confirm that $\alpha_i \neq 0$ and $\beta_i \neq 0$, as well as to confirm the quality of the model fit. Such a single-factor model may easily be represented as a graph – see Figure 8.1. The “abnormal” return denotes the portion of the return not related to factor $F$, as opposed to the “normal” return.

Note that the model is silent about what factor $F$ represents. Whatever the factor may be, it merely asserts that there exists an approximate linear relationship between factor $F$ and the rate of return on each fund – see Figure 8.2. Stated differently, fund returns are only related to each other through their common relationship with the factor.

Single-factor models provide a simple but effective framework for understanding and predicting returns. Other things being equal, a 1% variation in the factor is expected to result in
a $\beta_i$% variation in the return of fund $i$. Hence, beta is really a factor exposure indicator. A higher beta means more reaction to factor movements, while a lower beta means less reaction. Knowledge of the future value of $F$ could be used to predict asset returns, albeit not perfectly given the presence of the random error term.

Single-factor models also provide a very simple framework for understanding and predicting risk. By construction, the three components of returns in Figure 8.1 are not correlated with each other. Calculating the variance of both sides of equation (8.1) and eliminating constant terms yields:

$$\sigma_i^2 = \beta_i^2 \sigma_F^2 + \sigma_{\epsilon_i}^2$$  (8.2)

which may be interpreted as:

Total fund variance = Factor-related variance + Specific variance

This risk decomposition is a very useful way of thinking about risk. However, it is entirely dependent on the assumption that $\text{Corr}[\epsilon_i; F] = 0$, which in turn depends very much on the validity of the model specification.

### 8.2.2 Example: the CAPM and the market model

The granddaddy of all single-factor models is the CAPM due to Sharpe (1964), for which he shared a Nobel Prize in 1990. It is by no means the only single-factor asset pricing model, nor is it necessarily the best, but it remains the most widely known and applied.

Rather than modeling rates of return, the CAPM considers risk premium, i.e. the expected excess return above the risk-free rate. Under a certain number of assumptions, the CAPM says that at equilibrium, the risk premium of fund $i$ should be linearly related to the market risk premium. Algebraically:

$$R_i - R_F = \alpha_i + \beta_i \cdot (R_M - R_F) + \epsilon_i$$  (8.3)

where $R_i$ is the return on fund $i$, $R_F$ is the risk-free rate and $R_M$ is the return on the market portfolio. This model is also called the market model, or the security market line. Intuitively, it simply says that the return on the $i$th fund is made up of three components (Figure 8.3):

- A “normal return”, which corresponds to the fair reward for the market risk to which the portfolio is exposed. This risk premium of a fund depends on both the risk premium of the market itself (measured by $R_M - R_F$) and the sensitivity of the fund to the factor (measured by $\beta_i$).
- An “abnormal return”, which is the value added by the manager, which can be positive, negative or nil. In practice, the abnormal return is often called the alpha ($\alpha_i$) of the fund. According to the CAPM, alpha should be zero. Active managers attempt to seek incremental returns with a positive alpha, while passive managers will simply try to track the normal return and display an alpha equal to zero.
- Some statistical noise, which corresponds to the residual return ($\epsilon_i$). The role of $\epsilon_i$ is to allow unexplained forces to affect randomly the rate of return.

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2 According to theory, the market portfolio should be the value-weighted portfolio of all available risky assets including financial assets, real assets and even human capital. However, since we cannot readily observe the returns on such a portfolio, a stock index (such as the S&P 500) is usually used in practice as a proxy measure of market return – in which case $R_M$ denotes the return on the index portfolio used as a proxy.
Excess return on fund $i$

\[ R_i \]

\[ \beta_i \]

\[ \alpha_i \] Abnormal return

\[ \epsilon_i \] Statistical noise

\[ R_i \text{ (expected return)} \]

\[ \beta_i \text{ (beta)} \]

\[ \alpha_i \text{ (abnormal return)} \]

\[ \epsilon_i \text{ (statistical noise)} \]

**Figure 8.3** The three sources of return on a risky asset in the market model

### 8.2.3 Application: the market model and hedge funds

From a theoretical point of view, the CAPM represents an almost perfect blend of elegance and simplicity. Beta is an intuitively appealing measure of risk, whether one takes it as an asset’s contribution to total societal risk or the part of risk that cannot be diversified away. From an empirical point of view, the model appears to be readily testable. Betas are easily estimated from standard time series regressions and a linear risk-return trade-off seems to be tailor-made for empirical testing.

Black *et al.* (1972), Blume and Friend (1973) and Fama and MacBeth (1973) produced the first extensive tests of the model and confirmed a remarkably linear relationship between beta and the monthly returns on US equities. This tended to support the CAPM, at least empirically. Later, however, Roll (1977) criticized empirical tests of the CAPM and made a number of relevant objections about its testability in general. More recently, a number of researchers have evidenced the fact that multi-factor models perform better than single-factor models in explaining stock returns. We will focus on these models in the next section.

On the hedge fund side, the application of the market model has produced mixed results. Remember that one of the raisons d’être of alternative assets is to be non-correlated with traditional equity markets. Hence, when estimating the market model for a hedge fund, we often obtain regressions that have a low explanatory power, particularly when we consider funds outside the long/short equity category. As an illustration, we have estimated equation (8.3) for the five funds we considered in Chapter 4. The results are shown in Table 8.1. We see clearly that Funds 3 and 5, which hold essentially long and short positions in US equities, have the highest $R^2$ (0.54 and 0.42, respectively) of our sample, and display a statistically significant positive exposure to the US stock market. Fund 2, which is also invested in long and short positions in US equities, has a statistically significant positive exposure to the US stock market, but this exposure only captures a limited part of its behavior, as illustrated by the low $R^2$ (0.21). Fund 1, which is a fund of funds essentially invested in long/short...

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3 Roll’s objections can be summarized as follows:

- The *ex ante* linear relation of risk and return cannot be questioned by tests using *ex post* data. *Ex ante* and *ex post* returns are different quantities.
- The tests of the CAPM are tautological and follow from the mathematics of the efficient set of portfolios (i.e. the set of portfolios that provide the best possible return for a given level of volatility). In fact, there will always exist a linear relationship between the expected return of an asset and its beta with respect to an efficient portfolio.
- The total composition of the real market portfolio – and not of the market index used as a proxy – would have to be known for a test of the theoretical model that includes it as a variable.
Table 8.1  The market model applied to hedge funds
(an asterisk indicates coefficients that are significantly different from zero at 95% confidence)

<table>
<thead>
<tr>
<th>Fund</th>
<th>$\alpha_i$</th>
<th>$\beta_i$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fund 1</td>
<td>0.36</td>
<td>0.19*</td>
<td>0.16</td>
</tr>
<tr>
<td>Fund 2</td>
<td>0.43</td>
<td>0.64*</td>
<td>0.21</td>
</tr>
<tr>
<td>Fund 3</td>
<td>0.54*</td>
<td>0.43*</td>
<td>0.54</td>
</tr>
<tr>
<td>Fund 4</td>
<td>0.59</td>
<td>0.06</td>
<td>0.02</td>
</tr>
<tr>
<td>Fund 5</td>
<td>4.09*</td>
<td>2.08*</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Table 8.2  The market model applied to the CSFB/Tremont hedge fund indices, January 1994 to July 2002
(an asterisk indicates coefficients that are significantly different from zero at 95% confidence)

<table>
<thead>
<tr>
<th>Strategy</th>
<th>$\alpha_i$</th>
<th>$\beta_i$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedge fund index</td>
<td>0.36</td>
<td>0.25*</td>
<td>0.22</td>
</tr>
<tr>
<td>Global macro</td>
<td>0.69*</td>
<td>0.18*</td>
<td>0.05</td>
</tr>
<tr>
<td>Managed futures</td>
<td>0.31</td>
<td>-0.19*</td>
<td>0.06</td>
</tr>
<tr>
<td>Long/short equity</td>
<td>0.37</td>
<td>0.4*</td>
<td>0.33</td>
</tr>
<tr>
<td>Emerging markets</td>
<td>-0.06</td>
<td>0.54*</td>
<td>0.23</td>
</tr>
<tr>
<td>Dedicated short bias</td>
<td>0.01</td>
<td>-0.87*</td>
<td>0.59</td>
</tr>
<tr>
<td>Event-driven: distressed</td>
<td>0.51*</td>
<td>0.24*</td>
<td>0.29</td>
</tr>
<tr>
<td>Event-driven: risk arbitrage</td>
<td>0.2</td>
<td>0.12*</td>
<td>0.18</td>
</tr>
<tr>
<td>Event-driven: multi-strategy</td>
<td>0.31</td>
<td>0.19*</td>
<td>0.22</td>
</tr>
<tr>
<td>Convertible arbitrage</td>
<td>0.41*</td>
<td>0.04</td>
<td>0.01</td>
</tr>
<tr>
<td>Fixed income arbitrage</td>
<td>0.14</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Equity market neutral</td>
<td>0.41*</td>
<td>0.08*</td>
<td>0.15</td>
</tr>
</tbody>
</table>

equity managers, presents the same type of results. Finally, Fund 4, which is an arbitrage fund, is almost unrelated to the market ($R^2$ equals 0.02) and has no statistically significant coefficient.

We repeated the same analysis on the CSFB/Tremont hedge fund indices. The results are summarized in Table 8.2. With the exception of dedicated short bias funds, all $R^2$ are lower than 0.50. Long/short equity, emerging market and event-driven are the only strategies where the market model still has a small explanatory power. For the arbitrage-type strategies, the market model seems irrelevant.

### 8.3 Linear Multi-Factor Models

It is utopian to believe that rates of return on all available hedge funds (and hence, their covariance) can be satisfactorily explained by a single factor, whatever that factor may be. Hedge fund managers have different investment styles and opportunities, trade in multiple markets, take long and short positions, and use varying degrees of leverage. Hence, for most applications, a single-factor model will be too restrictive and will do a poor job at explaining hedge fund returns. Intuitively, there is more than one systematic risk factor in the picture. This is precisely where multi-factor models come in.
8.3.1 Multi-factor models

Multi-factor models can be viewed as a natural extension of single-factor models. Simply stated, linear multi-factor models postulate that rates of return on all available funds (and hence, their covariance) are explained by a linear function of several variables, the so-called “factors”. In its most generic form, a $K$-factor model can be written as:

$$ R_i = \alpha_i + \beta_{i,1} \cdot F_1 + \beta_{i,2} \cdot F_2 + \cdots + \beta_{i,K} \cdot F_K + \epsilon_i $$

(8.4)

or equivalently:

$$ R_i = \alpha_i + \sum_{j=1}^{K} \beta_{i,j} \cdot F_j + \epsilon_i $$

(8.5)

where $R_i$ represents the return on the $i$th fund, $K > 0$ is the number of factors, $F_1, \ldots, F_K$ are the values of the factors, $\beta_{i,1}, \ldots, \beta_{i,K}$ are the corresponding sensitivities, and $\epsilon_i$ is a zero mean random variable (Figure 8.4). The particular case of $K = 1$ corresponds to the single-factor model we saw previously.

Moving from a single- to a multiple-factor asset pricing model is conceptually easy and intellectually appealing. Most of the results from the single-factor case carry over to multi-factor models – only the notation becomes a little messier. However, the shift raises two important questions. First, how many factors are to be considered? Second, how should we determine these factors? The answers to these two questions are intimately linked and depend on the methodology used to build the factor model. In practice, there are essentially two main approaches used to identify factors, namely, principal component analysis (PCA) and common factor analysis (CFA).

8.3.2 Principal component analysis

Principal component analysis is a well-established statistical technique for exploring the (unknown) nature of the factor structure that is hidden behind some data set. It has applications in several fields such as data compression, visualization, exploratory data analysis, pattern recognition, time series prediction and of course finance. In this section, we do not examine in detail the computational aspects of PCA, but rather provide an intuitive introduction to this important technique.\footnote{See Kendall (1980).}
8.3.2.1 A primer on PCA

From a mathematical standpoint, the goal of PCA is to explain the behavior of a number of correlated variables using a smaller number of uncorrelated and unobserved implied variables or implicit factors called principal components. Each implicit factor is defined as a linear combination of original variables, so that the set of principal components reproduces the original information present in the correlation structure. That is, the first principal component is the linear combination of the original variables that accounts for as much variation in the data as possible. Each succeeding principal component is the new linear combination of the original variables that accounts for as much of the remaining unexplained variation as possible, and is independent of the preceding principal components.

As an illustration, say we have a sample of \( N \) hedge funds and \( P \) measurable parameters for each of them. Examples of such parameters are the assets under management, the time since inception, the average return and the volatility over the last three years, the returns over a particular number of months, etc. Now, say \( N \) and \( P \) are so large that it is unwieldy and impractical to analyze how these \( P \) parameters are related to each other. We obviously cannot look at all the possible correlations and scatter plots, because of the excessive number of parameters and funds. We would therefore like some objective means of reducing the variables to a manageable number, while at the same time preserving as much of the variability in the data as possible. This is precisely where PCA comes into action. Basically, PCA looks for sets of parameters that are always highly correlated. Then PCA creates a new parameter by grouping (linear combination) these correlated parameters, therefore reducing the dimensionality of parameters without losing too much information. One drawback of the technique is that the new parameters do not have a direct economic interpretation.

One aspect that is often scrutinized is the number of implied factors that are relevant in PCA. At most, there can be as many possible principal components as there are variables. That is, to reproduce the total system variability of the original \( N \) variables, we need \( N \) principal components. However, we have to remember that principal components are extracted in decreasing order of importance. Consequently, the first few principal components usually account for a large proportion of the variance in the sample. In fact, since principal components are chosen solely for their ability to explain risk, a given number of implicit factors always capture a larger part of the asset return variance–covariance than the same number of explicit factors. Therefore, a standard approach to deciding when to stop extracting principal components is to generate a scree plot as suggested by Cattell (1966). The scree plot is a two-dimensional graph with all components on the \( x \)-axis and eigenvalues on the \( y \)-axis. Eigenvalues represent the variance accounted for by each principal component expressed as a score that totals the number of items.\(^5\) The eigenvalues are typically arranged in a scree plot in descending order like in Figure 8.5.

From the scree plot you can see that the first couple of factors account for most of the variance, then the remaining factors all have small eigenvalues. You can thus choose the number of principal components to use depending on how much variance you want to explain. Therefore selecting the number of factors involves a certain amount of subjective judgment.\(^6\)

\(^5\) That is, the total of all the eigenvalues will be \( N \) if there are \( N \) items in the data sample, so some factors will have smaller eigenvalues. If the first factor has an eigenvalue of \( N_1 \), it accounts for \( N_1/N\% \) of the variance of the data sample.

\(^6\) Another approach is called the Kaiser–Guttman rule and simply states that the number of principal components is equal to the number of factors with eigenvalues greater than 1.0. An eigenvalue greater than 1.0 indicates that principal components account for more variance than accounted for by one of the original variables in standardized data. This is commonly used as a cut-off point for which principal components are retained.
Figure 8.5 The scree plot for the CSFB/Tremont indices. For the sake of simplicity, we have only represented the first 10 components.

Figure 8.5 shows the scree plot that we obtained when applying PCA to the time series of returns of the 12 CSFB/Tremont hedge fund indices since their creation. The scree plot indicates that the 12 indices (which cover all investment styles) share a common component, which explains 55% of the variance of the data sample. There is also a second common component that is orthogonal to the first component (by construction) and explains an additional 18% of the variance. Together, the first and second components explain 73% of what is happening. Adding a third component would raise the explanatory power to 83.2%, etc. In total, we could go up to 12 components and explain 100% of the variance.

Once the number of factors is chosen, the next step is the extraction of the principal components from the considered data sample. As mentioned already, these principal components are linear functions of the original variables.\footnote{The weights to compute the uncorrelated principal components are called eigenvectors.} In the CSFB/Tremont index, the first principal component is defined as

\[
\text{PCA}_1 = (0.253 \times \text{Tremont general}) + (0.219 \times \text{Tremont global macro}) \\
+ (0.347 \times \text{Tremont long/short equity}) + (0.584 \times \text{Tremont emerging markets}) \\
- (0.571 \times \text{Tremont dedicated short}) + (0.197 \times \text{Tremont distressed}) \\
+ (0.189 \times \text{Tremont multi-strategy})
\]

Although it may seem hard to identify what PCA$_1$ effectively represents, a rapid correlation analysis evidences that it has a correlation of 0.71 with the MSCI World and 0.75 with the Nasdaq Composite. It is therefore likely to represent some sort of equity market indicator. A similar analysis can then be applied to the second component, the third, etc.

To understand what these components represent, it is useful to use a geometric representation. Consider again the sample of $N$ hedge funds and $P$ measurable characteristics. We can
represent the $N$ funds by a large cloud in a $P$-dimensional space. If two or more characteristics are correlated, the cloud will be elongated along some direction in the $P$-dimensional hyperspace defined by their axes. PCA will identify this extended direction and form the first principal component. Next, we consider the $(P - 1)$-dimensional hyperplane orthogonal to the first component and search for the direction in the $(P - 1)$-space that represents the greatest variability. This defines the second principal component. The process is continued, defining a total of $P$ orthogonal directions.

As an illustration, Figure 8.6 shows what happens with only two measurable characteristics ($N = 22$, $P = 2$). The first component (PCA$_1$) can be seen as a sort of regression line, which gives an optimal (in the mean square sense) linear reduction of dimension from two to one dimensions. The second component (PCA$_2$) is orthogonal to PCA$_1$. Note that this graph also illustrates the major drawback of PCA, i.e. the extreme difficulty in interpreting what the principal components (i.e. the new axes PCA$_1$ and PCA$_2$) effectively represent from an economic perspective.

The final step of PCA is usually the rotation of the original axes. For example, in our scatter plot we can think of the regression line as the original $x$-axis, rotated so that it approximates the regression line. This type of rotation is called variance maximizing because the criterion for (goal of) the rotation is to maximize the variance (variability) of the “new” variable (factor) while minimizing the variance around the new variable. In general, the idea is to rotate the axes in a more convenient direction without changing the relative locations of the points to each other.

The popularity of PCA comes essentially from two properties. First, PCA is the optimal (in terms of mean squared error) linear scheme for compressing a set of high-dimensional variables into a set of lower-dimensional variables and then reconstructing. Second, the principal components can be constructed directly using either original multivariate data sets or using the covariance or the correlation matrix if the original data set is not available.\(\text{8}\)

---

\(\text{8}\) The correlation matrix is commonly used when different variables in the data set are measured using different units or if different variables have different variances. In a sense, it is equivalent to standardizing the variables to zero mean and unit standard deviation. This standardization is recommended for hedge funds because of their different degrees of leverage. After standardization, hedge fund returns can be compared irrespective of different degrees of leverage.
8.3.2.2 Application to hedge funds

Fung and Hsieh (1997a) were the first authors to use PCA to extract a set of implicit factors for hedge funds and to provide a quantitative classification scheme based on returns alone. Their idea is simply that if a group of managers follow the same strategy on the same markets, their returns should be correlated to each other, even though they may not be linearly correlated to the returns of asset markets. The advantage of using PCA in such a case is that it subsumes the inherent non-linearity of hedge fund returns and easily identifies the set of common factors.

Using a database provided by TASS Asset Management, AIG Global Investors and Paradigm LDC, Fung and Hsieh found that the first five principal components jointly explain about 43% of the cross-sectional variation in hedge fund returns. The good news is that by analyzing the qualitative self-description of the respective strategies of the funds that are correlated with the first five principal components, Fung and Hsieh were able to assign meaningful names to each of these principal components. The first component consists of funds applying a trend-following strategy on diversified markets, i.e. managed futures and commodity trading advisors. The second component represents global/macro funds. The third component is made up of long/short equity funds. The fourth component corresponds to funds that apply a trend-following style with an emphasis on major currencies. The fifth component is made up of distressed securities funds. Interestingly, the majority of market neutral and non-directional funds fall into the unexplained category in the principal component analysis.

More recently, Christiansen et al. (2003) applied a similar approach to the Zurich Alternative Investment Performance Database/CISDM (the former MAR database). They also identified five orthogonal components, but were able to explain more than 60% of the cross-sectional variation in hedge fund returns. The first component explains 24.92% of the variance (versus 11.87% in Fung and Hsieh) and essentially corresponds to long/short equity strategies. The remaining four components explain 12.65%, 10.80%, 7.04% and 4.65% of the variance versus 10.00%, 9.42%, 6.35% and 4.93%, respectively in the Fung and Hsieh analysis. The second component is identified as event-driven strategies, the third component is global macro and the fourth component is dominated by market neutral strategies mainly based on long/short US equity. Finally, the fifth component regroups funds active in the relative value arbitrage.

Amenc and Martellini (2001b, 2003a) suggested another extremely interesting application of passive hedge fund index or “index of the indexes indices”. We already briefly introduced their methodology in Chapter 5. In fact, their approach is a natural generalization of the idea of taking an equally weighted portfolio of competing indices. Namely, they are looking for a portfolio of competing indices, where the portfolio weights make the combination of indices capture the largest possible fraction of the information contained in the data from the various competing indices. Technically speaking, this amounts to using the first component of a PCA of competing indices as a candidate for a pure style index. This first component typically captures a large proportion of cross-sectional variations because competing indices tend to be at least somewhat positively correlated. By construction, the resulting indices (one for each of the 13 strategies) have a higher degree of representation and stability of hedge fund performance than the individual indices already available on the market.

8.3.3 Common factor analysis

Common factor analysis is usually what people mean when they say factor analysis. The goal of common factor analysis is similar to PCA, that is, to transform a number of correlated
variables into a smaller number of uncorrelated variables called \textit{factors}. However, unlike PCA, the selected factors are observable and explicitly specified by a mix of explanatory and/or confirmatory analyses\textsuperscript{9} rather than implied by the data. The number of factors, which should be kept as small as possible to maximize the benefits of dimensionality reduction, is often a trade-off between goodness-of-fit and overfitting. Most of the time, this crucial parameter is also assumed rather than being determined by the data.\textsuperscript{10}

\subsection*{8.3.3.1 Examples of typical factors}

Very often, with common factor analysis, the final choice of factors is done on an ad hoc basis. The standard criterion is to choose variables that are thought most likely to influence asset returns. Here again, quantitative analysts have a lot to gain by communicating with qualitative analysts, who talk to fund managers and may have relevant suggestions about which factors to use. Of course, quantitative analysts should also look at the empirical asset pricing literature. For many years, academics have been hunting for factors that would help explain the cross-section of expected returns. For instance, among the usual candidates for stock portfolios are the market value of equity or market capitalization (size) suggested by Banz (1981) and Reinganum (1981), the book-to-market ratio suggested by Stattman (1980), the leverage suggested by Bhandari (1988), the earnings-to-price ratio suggested by Basu (1983) and the stock liquidity suggested by Amihud (2002), among others. Recognizing that some of these factors are highly correlated (e.g. book-to-market ratio and leverage), several academics have suggested parsimonious extensions to the original market model. We will review two of them in greater detail, because they have shaped asset pricing research in recent years. One is Fama and French’s (1992, 1996) research on the size and book-to-market factors, and the other is Jegadeesh and Titman’s (1993) and Carhart’s (1997) research on the momentum factor.

\subsection*{8.3.3.2 Size and book-to-market}

Eugene Fama and Kenneth French, two professors at the University of Chicago, published a series of empirical studies that historically dealt the most damaging blow to both the CAPM and single-factor models. Fama and French monitored 9500 US stocks from 1963 to 1990, and observed that two classes of stocks tended to do better on average than the market as a whole: (i) small caps and (ii) stocks with a high book value-to-price ratio, customarily called “value stocks” in contrast to “growth stocks”. Hence, Fama and French suggested including size and book-to-market ratio as remunerated risk factors to obtain a three-factor asset pricing model. Their model is:

\begin{equation}
E(R_i) = R_F + \beta_{i,1}[E(R_M) - R_F] + \beta_{i,2} \cdot E(SMB) + \beta_{i,3} \cdot E(HML) \quad (8.6)
\end{equation}

or, in an \textit{ex post} form:

\begin{equation}
R_i - R_F = \alpha_i + \beta_{i,1}[R_M - R_F] + \beta_{i,2} \cdot SMB + \beta_{i,3} \cdot HML + \epsilon_i \quad (8.7)
\end{equation}

where $R_i$ is the expected return on asset $i$, $R_F$ is the return on the risk-free asset and $R_M$ is the expected return on the market portfolio, SMB is the return on the size factor and HML

\textsuperscript{9} In explanatory analysis, several factors are tested, with the hope that some of them will explain well the data being analyzed. A typical technique consists in screening hundreds of potential factors using stepwise regression. In confirmatory analysis, an economic model is formulated \textit{a priori} (before seeing the data) and then tested on a given data set.

\textsuperscript{10} Note that there exist econometric techniques to identify the optimal number of factors. See for instance Bai and Ng (2002).
Figure 8.7 The performance of $100 invested in the SMB portfolio on 1 January 1990

is the return on the book-to-market factor. SMB is a zero-investment portfolio that is long small capitalization stocks and short big capitalization stocks (Figure 8.7).\(^{11}\) HML is a zero-investment portfolio that is long high book-to-market stocks and short low book-to-market stocks (Figure 8.8). The parameter \(\beta_{i,2}\) measures asset \(i\)'s sensitivity to the size factor, and the parameter \(\beta_{i,3}\) measures asset \(i\)'s sensitivity to the book-to-market factor.

It is important to realize that the Fama and French model is not an equilibrium model. It is purely empirically motivated, and there is no theory telling us what gives rise to the SMB and HML factors.\(^{12}\) In a sense, Fama and French looked at the data and chose factors based on what they found there. Nevertheless, several empirical tests evidenced that, when considering SMB and HML as additional factors, beta was no longer a reliable predictor of performance. This led Eugene Fama to declare that “beta as the sole variable in explaining returns on stocks . . . is dead”.

Box 8.1 How to build the HML and SMB factors

In linear factor models, it is extremely convenient to use factors that correspond to the returns on a given observable portfolio. If the factor considered is an abstract concept that is not necessarily observable, we use a portfolio of assets whose returns are highly correlated with the (unobservable) factor values. The returns to the factor-mimicking portfolios mimic the factor values in a certain sense – remember that the factor values themselves

\(^{11}\) The portfolio does not need any investment because the cash received from short selling stocks is used to finance the purchase of other stocks.

\(^{12}\) The Fama and French findings imply that stocks with a high book-to-market ratio must be more risky than average – exactly the opposite of what a traditional business analyst would claim. However, there are several possible explanations for this phenomenon, e.g. a high book-to-price ratio could mean a stock is distressed, temporarily selling low because future earnings look doubtful. Or it could mean a stock is capital intensive, making it generally more vulnerable to low earnings during slow economic times. Black (1993) also pointed out that when markets are somewhat efficient, stock prices react to a firm’s performance before accounting numbers. Thus it is not surprising that firms with a high book-to-market ratio show poor subsequent accounting performance.
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are usually not directly observable. This is precisely the situation with the SMB and HML portfolios.

Fama and French’s method for creating the HML and SMB portfolios is as follows. First, sort stocks simultaneously using two independent sorts, one on the market capitalization (small/big) and one on the book-to-market ratio (high/medium/low). Using the sorted universe, create six portfolios labeled small high (SH), small medium (SM), small low (SL), big high (BH), big medium (BM) and big low (BL). The size breakpoint that determines the buy range for the small and big portfolios is the median NYSE market equity. The book-to-market breakpoints that determine the buy range for the growth, neutral and value portfolios are the 30th and 70th NYSE percentiles.

Then, the HML and SMB portfolios are calculated as:

\[ HML = \left( \frac{1}{2} SH + \frac{1}{2} BH \right) - \left( \frac{1}{2} SL + \frac{1}{2} BL \right) \]  

and

\[ SMB = \left( \frac{1}{3} SH + \frac{1}{3} SM + \frac{1}{3} SL \right) - \left( \frac{1}{3} BH + \frac{1}{3} BM + \frac{1}{3} BL \right) \]

Although SMB and HML are zero-investment portfolios, they earn positive returns, as illustrated in Figures 8.7 and 8.8. Fama and French argue that HML and SMB are state variables that describe changes in the investment opportunity set. The stocks considered each month to construct the HML and SMB factors include all NYSE, AMEX and NASDAQ stocks with prior return data. Note that the website of Kenneth French provides monthly updates on the value of these two factors.

8.3.3.3 Price momentum

Another essential piece of research suggesting the addition of new factors to the market model was that of Jegadeesh and Titman (1993). The two authors documented the existence of a
Momentum effect: strategies to buy stocks that have performed relatively well in the past and sell short stocks that have performed relatively poorly in the past generate significant positive returns over a 3–12-month horizon. These return continuation strategies have been tested extensively and confirmed on several markets.13

Following Carhart (1997), a large number of researchers have therefore suggested adding a momentum factor, WML (winners minus losers) to the Fama and French three-factor model. This gives a four-factor pricing model that explains the excess return of a security by the market portfolio and three factors designed to mimic risk variables related to size, book-to-market ratio and momentum:

$$R_i - R_F = \alpha_i + \beta_{i,1} (R_M - R_F) + \beta_{i,2} \cdot SMB + \beta_{i,3} \cdot HML + \beta_{i,4} \cdot WML + \epsilon_i$$  \hspace{1cm} (8.10)

where WML is the return on the momentum factor. WML is basically a zero-investment portfolio that is long past winners and short past losers (Figure 8.9).

The returns of the winner-minus-loser momentum portfolio are particularly impressive. However, we should note that they are mostly due to the poor performance of the losers. So, in order to capture the bulk of the momentum effect, short positions are necessary. In practice, holding the WML factor involves a very high turnover, and transaction costs and taxes may significantly erode momentum profits. Furthermore, the momentum effect is stronger among small cap stocks, which tend to be less liquid. Trying to implement a high-turnover strategy with small cap stocks is unrealistic. Hence the actual returns from a winner-minus-loser momentum portfolio are likely to be much lower in practice.

Box 8.2 How to build the momentum factor

The construction of the momentum factor parallels the calculation of Fama and French’s SMB and HML factors.

For each month \( t \) from July of year \( Y - 1 \) to June of year \( Y \), we rank the stocks based on their size in June \( Y - 1 \) and their performance between \( t - 12 \) and \( t - 2 \). We then use these two rankings to calculate 30% and 70% breakpoints for prior performance. The stocks are subsequently sorted into three prior performance groups based on these breakpoints. The stocks above the 70% prior performance breakpoint are designated W (for winner), the middle 40 are designated N (for neutral) and the firms below the 30% book-to-market breakpoint are designated L (for loser). The WML portfolio is the difference between the top 70% stocks and the bottom 30% – the neutrals are simply ignored. The stocks considered each month to construct the WML portfolio include NYSE, AMEX and NASDAQ stocks with prior return data. Note that the website of Kenneth French provides regular updates on the value of the WML factor.

8.3.3.4 Other market-based factors

The debate on whether the Fama and French three-factor model or the Carhart four-factor model explain well the economic risk of traditional assets is still ongoing, but it is not our primary concern here. In the domain of hedge funds, the challenge is still to identify a small number of economically interpretable factors that can capture the majority of the underlying risk for all the strategies.

There are many reasons to believe that the sources of risk in hedge funds should not be very different from the sources of risk in traditional assets. At the end of the day, both types of managers are actually investing in the same markets, probably even buying and selling the same stocks and bonds. The type of trades (buying versus selling) and the way these assets are managed, however, are different. Therefore, while many hedge fund investors believe that investing in hedge funds is all about the “search for alpha”, a deeper analysis of hedge funds reveals that the “search for alpha” has to first start with an “understanding of beta”.

Several factor models have been suggested in the literature over recent years – see for instance Schneeweis and Spurgin (1998), Schneeweis et al. (2001), Capocci (2001), Amenc et al. (2002a) or Amenc et al. (2002b), among others. Reading these studies provides a good overview of the potential return drivers, i.e. the factors that are likely to explain a significant proportion of the variation in hedge fund returns over time. Most of these return drivers are fundamentally associated with the underlying holdings of the strategy and the strategy itself. As an illustration, we have listed some of these factors, as well as the typical indicator/index associated with each of them. Note that for many of these factors, it is interesting to distinguish the absolute level of the factor from its percentage change.

**Equity-related factors**

- Market indices (S&P 500, MSCI World, etc.)
- Sectors (MSCI World sector indices)
- Traded volume (NYSE cumulated volume)
Equity trading styles
- Value versus growth (Fama and French factors)
- Small versus big capitalization (Fama and French factors)
- Momentum (Fama and French factors)

Interest rate factors
- T-bill rate (3-month Treasury rate)
- Slope of the yield curve (30-year Treasury yield minus three-month T-bill yield)
- Credit risk premium (difference between BBB and AAA yield)

Currencies and commodities
- Selected basket of exchange rates (USD/EUR, USD/JPY, etc.)
- Selected basket of commodities (gold, oil, etc.)

Stability-related factors
- Implied volatility index (VIX for options on S&P 100 index)
- Intra-month volatility of bond returns (intra-month standard deviation of the daily total rate of Lehman Brothers Aggregate Bond index)
- Intra-month volatility of equity returns (intra-month standard deviation of S&P 500 index)

Others
- Confidence index
- GDP growth
- etc.

In addition to these market-related factors, we may also include in our factor models some characteristics that are specific to a given fund rather than to a particular strategy or an investment style. For instance, fund size and fund age are likely to affect hedge fund performance in different ways, as illustrated in Figures 8.10 and 8.11. We see clearly that for bond arbitrage and market neutral, performance (i.e. Sharpe ratio) degrades rapidly as fund size is increased, while this effect is not as dramatic or reversed for other strategies. Convertible arbitrage and distressed securities fund performances even appear to improve with fund size. We also see that for most strategies, the younger funds have significantly better Sharpe ratios than the older funds, perhaps due to size and nimbleness. These results should be taken with caution, because younger funds are likely to be subject to the greatest reporting bias.

8.3.4 How useful are multi-factor models?
There are several domains of application for hedge fund multi-factor models. Among others, let us quote the following:
- The identification of the relevant drivers of performance of a portfolio of funds or a hedge fund index.
Figure 8.10  Sharpe ratios of equally weighted portfolios stratified by size. Small means less than $25 million of assets under management and large more than $200 million. Note that some categories are empty.

Figure 8.11  Sharpe ratios of equally weighted portfolios stratified by age. Baby means less than a year of existence, Young means 1 to 2 years, Adolescent means 2 to 3 years, Adult means 3 to 5 years and Mature means more than 5 years. Note that some categories are empty.
The asset pricing models provide an interesting and natural explanation for the change in correlations observed between hedge funds. Correlation will be higher when systematic macroeconomic factors, which affect all assets in tandem, dominate fund-specific factors. If variations in asset returns are driven by both systematic factors and idiosyncratic (i.e., sector-specific) risks, then periods of high factor volatility will coincide with periods of high correlation. During these periods, the dominant source of variation will be due to the common factor. Unless idiosyncratic volatility is strongly correlated across sectors (which is not the case in our sample), periods of high cross-sector correlation will coincide with periods of high overall market volatility. Thus, correlation breakdown—the strong observed association between correlation and volatility—is not simply bad luck. It is to be expected. Moreover, it is not evidence that the structure of security returns periodically changes perversely; “breakdown” can be part and parcel of a stable factor model of returns.

### Box 8.3 When lagged returns may help

As already mentioned in Chapter 5, hedge fund managers can smooth portfolio returns when marking illiquid securities using historical prices (e.g., last traded price, hence stale) or worse, a price that the manager thinks is reasonable. Pricing of these securities is thus stale or managed, and this potentially leads to understating volatility as well as betas and correlations with traditional indexes.

To take into account this phenomenon, Asness et al. (2001) suggest running a regression of returns on both contemporaneous and lagged market returns, and testing whether lagged betas are significant.

Using the CSFB/Tremont hedge fund index’s monthly returns between January 1994 and September 2000, Asness et al. observe that (i) lagged betas are significant for every hedge fund style except managed futures and (ii) including lagged betas in the regression significantly reduces the alpha. Asness et al. also suspect some evidence of intentional stale prices, because the summed lagged beta in down markets is statistically significant only in down markets.

### 8.4 ACCOUNTING FOR NON-LINEARITY

Almost all the traditional asset pricing models, including the single-factor and multi-factor models that we have considered so far, are based on the assumption that the expected return on a hedge fund is a linear function of some common factors.

A first key assumption underlying such models is that volatility is the appropriate measure of risk to the investor. It is then a mathematical consequence of this that only systematic risk
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(i.e. the covariance\(^{14}\)) is rewarded in financial markets. However, if this assumption seems quite reasonable for assets with returns that are symmetrically distributed, it does not seem so reasonable for hedge funds whose returns are asymmetrically distributed.\(^{15}\) In such a case, investors will care about other aspects of the distribution, such as the skewness, kurtosis, etc.

A second key assumption underlying traditional asset pricing models is that risk exposures (i.e. beta coefficients) are constant through time. However, this is unlikely to be true for hedge funds, whose managers follow highly opportunistic strategies and have time-varying risk exposures. A large body of literature has recently documented that asset returns were to some extent predictable on the basis of proper instrument variables, and this modifies the constriction of portfolios in at least three ways. First, it introduces horizon effects, as mean and variance may no longer scale the same way with horizon – see Barberis (2000). Second, it allows market timing strategies. Portfolio managers use information about the current state of the economy to form return and risk expectations and adjust their portfolios accordingly. Third, it introduces multiple factors via hedging demands – if expected returns vary over time, investors will hold assets that protect them against this risk. Given all these changes, traditional static asset pricing models may be incorrectly specified if we do not allow for time variation in the risk exposures.

These new facts have attracted academic attention and led to a new series of asset pricing models. We briefly review them in this section and provide essential references for those readers who would like to investigate them in further detail.

8.4.1 Introducing higher moments: co-skewness and co-kurtosis

Investors are concerned about risk, and risk must be measured in terms of the entire probability distribution, which in turn can be measured with the moments of the distribution. Thus, several authors have posited that investors display a preference for positive skewness and prefer assets that are right-skewed to assets that are left-skewed – see for instance Rubinstein (1973), Kraus and Litzenberger (1976) or Harvey and Siddique (2000). If this is true, then investors should value the skewness contributions to their portfolios, or more precisely, the co-skewness of underlying assets. That is, assets that decrease a portfolio’s skewness (i.e. that make the portfolio returns more left-skewed) are less desirable and should command higher expected returns. Similarly, assets that increase a portfolio’s skewness should have lower expected returns. Note that it is only the co-skewness that should count, not the skewness alone.\(^{16}\)

A similar case can be developed for kurtosis and co-kurtosis, and more generally, for any higher-order moment of the return distribution. In the general case, Scott and Horvath (1980) show that investors should have a negative preference for even moments and a positive preference for odd ones. Consequently, these moments should be considered in an asset pricing model. Only in very special and unlikely cases, such as quadratic utility or normality of returns, can we ignore the higher moments and focus on just mean and variance (or covariance).

Rubinstein (1973) derives an equation for the expected return in terms of an arbitrary number of co-moments. In his model, the expected return on an asset should be equal to the risk-free

\(^{14}\) Remember that the variance of a portfolio is built from the covariance of its constituting assets.

\(^{15}\) Consider, for instance, these two lotteries: the first costs one dollar, and there is one chance in 10 million of winning one million dollars; the second pays one dollar upfront, but there exists one chance in a million of having to pay one million dollars. Returns from both lotteries have the same mean, variance and even moments. Which one looks more attractive? Most people would prefer the first lottery.

\(^{16}\) This is analogous to beta and variance. In the CAPM, it is only the beta that is rewarded, not the total volatility.
rate, plus a risk premium equal to a weighted sum of all co-moments of the return distribution. In particular, if investors only care about the first three moments, we obtain a three-moment CAPM:

\[
E(R_i) - R_F = \beta_i \left( \frac{(E(R_i) - R_F)(E(R_M) - R_F)}{(E(R_M) - R_F)^2} \right) + \gamma_i \left( \frac{(E(R_i) - R_F)(E(R_M) - R_F)^2}{(E(R_M) - R_F)^3} \right)
\]

(8.11)

Kraus and Litzenberger (1976) derive an equation of the same form and evidence that this three-moment CAPM is consistent with a quadratic return generating process of the form

\[
R_i - R_F = \alpha_i + \beta_{i,1}[R_M - R_F] + \beta_{i,2}[R_M - R_F]^2 + \epsilon_i
\]

(8.12)

This provides a convenient way to test for skewness preferences in asset pricing models, as we just need to test whether \(\beta_{i,2}\) is statistically significant. Kraus and Litzenberger also state “it is trivial to extend the model to incorporate any number of higher moments”. For example, if we assume identical investors, we can derive a four-moment CAPM where all four moments – mean, systematic variance, systematic skewness and systematic kurtosis – contribute to the risk premium of an asset. This corresponds to a cubic return generating process of the form

\[
R_i - R_F = \alpha_i + \beta_{i,1}[R_M - R_F] + \beta_{i,2}[R_M - R_F]^2 + \beta_{i,3}[R_M - R_F]^3 + \epsilon_i
\]

(8.13)

As underlined by Barone-Adesi et al. (2002), the cubic model does not allow for a precise estimation of the co-skewness and co-kurtosis risk premiums, but it provides a powerful test of the relationship between risk and expected return implied by the asset pricing model.

**Box 8.4 From market timing to co-skewness pricing**

There is an interesting parallel between the quadratic asset pricing model and the Treynor and Mazuy (1966) approach to measure market timing. In its pure form, market timing involves shifting funds between a market index and a risk-free asset (cash), depending on whether the market as a whole is expected to outperform the risk-free asset. If a fund manager holds constant the relative proportion between cash and the market index, his fund’s beta will be constant and his returns will plot along the security market line. But if he successfully engages in market timing activities, he will increase his market exposure on the upside and decrease it on the downside, thereby altering the linear security market line of the single-factor model. The new relationship should be curvilinear, portfolio return becoming a convex function of market return. Similarly, bad timing activities will result in a concave relationship. Therefore, to test for market timing, Treynor and Mazuy (1966) propose to add a quadratic term to CAPM, that is:

\[
R_i - R_F = \alpha_i + \beta_{i,1}[R_M - R_F] + \beta_{i,2}[R_M - R_F]^2 + \epsilon_i
\]

(8.14)

where \(\beta_{i,2} > 0\) indicates successful market timing ability.\(^{17}\) This timing model is exactly the same as the quadratic asset pricing model used to include skewness preference.

\(^{17}\) A formal treatment of this test can be found in Admati et al. (1986). Note that \(\beta_{i,2}\) should not be used to rank market timers, since it combines both the quality of the manager’s private information and his response to this information (his “aggressiveness”).
Ranaldo and Favre (2003) have applied the quadratic and cubic asset pricing models. Their results are mixed. For eight out of 16 hedge fund strategies investigated, the coefficients of co-skewness and co-kurtosis are not significant. For the eight other strategies, the coefficients of co-skewness and co-kurtosis are significant, and therefore risk exposure should be compensated. Hence, the two-moment CAPM tends to underestimate the required rate of return for these hedge funds.

A very interesting observation by Chung et al. (2001) is that SMB and HML generally become insignificant or much less significant as systematic co-moments are added into the picture. This tends to suggest that Fama–French factors may simply be good proxies for higher co-moments of the return distribution. Given the econometric problems of estimating co-moments, SMB and HML could be superior in actual use.

### 8.4.2 Conditional approaches

Whatever their level of sophistication and number of factors, unconditional asset pricing models such as the ones we have investigated so far may provide incorrect conclusions when fund managers react to market information or engage in dynamic trading strategies – see Ferson and Schadt (1996) and Ferson and Warther (1996). For the sake of illustration, let us consider a one-factor model similar to the one introduced in equation (8.3). Let us assume that the manager of fund \( i \) engages in market timing activities, that is, the beta of his portfolio is \( \beta_{\text{up}} \) when the fund manager forecasts a positive performance and \( \beta_{\text{down}} \) otherwise.

The unconditional model is:

\[
R_i - R_F = \alpha_i + \beta_i \cdot (R_M - R_F) + \varepsilon_i \tag{8.15}
\]

where \( R_i \) is the return on fund \( i \), \( R_F \) is the risk-free rate and \( R_M \) is the return on the market portfolio. Equation (8.15) assumes that \( \alpha_i \) and \( \beta_i \) are constant over time. Since this is not true, the regression corresponding to equation (8.15) may provide little or no useful information about the true values of \( \alpha \) and \( \beta \). As an illustration, Figure 8.12 illustrates the risk of using unconditional models when the returns (and thus the betas) are state dependent. The fund manager has no alpha (\( \alpha = 0 \)) and clearly uses a bull market exposure (\( \beta_{\text{up}} \)) different from his down-market exposure (\( \beta_{\text{down}} \)). By contrast, an unconditional model would have concluded that the manager has an average beta and a positive alpha!

![Figure 8.12](image)

*Figure 8.12* The mis-estimation of beta creates alpha when the manager does market timing
Several alternatives have been used in the financial literature to model time-varying returns and risk. The simplest assumption, of course, is that there is a linear relationship between the coefficients at time $t$ and the coefficients at time $t - 1$. If we denote by $Z_{t - 1}$ the value at time $t - 1$ of the parameter that influences the alpha and beta coefficients of fund $i$ at time $t$, we have:

$$\begin{align*}
\alpha_{t,i} &= \bar{\alpha}_i + a_i Z_{t-1} \\
\beta_{t,i} &= \bar{\beta}_i + b_i Z_{t-1}
\end{align*}$$

(8.16)

where $\bar{\alpha}_i$ is the average abnormal performance of the portfolio, $\bar{\beta}_i$ is the average risk exposure of the portfolio, and $a_i$ and $b_i$ need to be estimated. The idea is that at time $t - 1$, the hedge fund manager observes $Z_{t-1}$ and adjusts his portfolio accordingly. Then, $a_i Z_{t-1}$ captures the time variation in the abnormal performance, i.e. the departure from the mean level of abnormal performance $\bar{\alpha}_i$. Similarly, $b_i Z_{t-1}$ captures the time variation in the risk exposure, i.e. the departure from the mean level of risk $\bar{\beta}_i$.

Then, applying the conditional model to equation (8.15), we arrive at

$$R_i - R_F = \alpha_i + \beta_i \cdot (R_M - R_F) + \beta_i \cdot (R_M - R_F)Z_{t-1} + \epsilon_i$$

(8.17)

This new regression can be viewed as an unconditional multiple-factor model, with excess return as the first factor and the cross-product of excess market return with the lagged information variable as the additional factor. It is possible to interpret the additional factor as the return to a dynamic strategy, which holds $Z_{t-1}$ units of the market index financed by borrowing $Z_{t-1}$ at the risk-free rate.

Note that in the model we have just seen:

- $Z_{t-1}$ denoted a single publicly available variable. In practice, several publicly available variables may influence the behavior of the alpha and beta coefficients. The conditional model will therefore express the excess return on the $i$th fund as a conditional linear function of the relevant lagged information variables at the beginning of the period.
- The model assumes a linear relationship between the regression parameters and a set of publicly available instruments. This idea was first introduced by Harvey (1989) and has been used extensively ever since – see for example Ferson and Harvey (1993).

Kat and Miffre (2002) provide an illustration of this technique for hedge funds using the Zurich Capital Markets database. Kat and Miffre compare three conditional-based models, namely, the conditional market model, the conditional Fama–French three-factor model and an explicit conditional macrofactor model that considers the market factor and five macroeconomic factors (exchange rate risk, term structure of interest rates, international risk of default on short maturity securities, inflation risk and industrial risk). Their results reject the hypothesis of constant parameters for about 79% of the funds for the market model, and for all the funds for the two multi-factor models. When considering the conditional six-factor model, the best predictor of abnormal hedge fund performance is own return (39% of the time). Next come the default spread (22.1%), the dividend yield (16.9%), the term structure (13%) and the Treasury bills (11.7%). The impact of these variables is – of course – different in periods of expansion and recession.

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18 Kazemi and Schneeweiss (2003) and Cerrahoglu et al. (2003) also use conditional approaches, but rely on a different technique called the stochastic discount factor.
8.5 HEDGE FUNDS AS OPTION PORTFOLIOS

For the public and the press, options and hedge funds share several anecdotal features. Both are perceived as complex, excessively leveraged and difficult to understand investments, except for specialists. Both were at the source of numerous media reports announcing spectacular gains and losses, which reinforced the view that all hedge fund managers and all derivative users were taking on risks that were far above average. Both have grown up outside of regular exchanges, in offshore countries for hedge funds and at over-the-counter desks for most active options. And both are easy to blame when there is a systemic market problem!

But the similarity between options and hedge funds is not just anecdotal. From an economic perspective, one can show that there exist several implicit option-related aspects in hedge funds. For instance:

• Hedge funds’ special fee structures aim at aligning managers’ incentives with fund performance. From a theoretical perspective, the incentive fee of hedge fund managers can be seen as a call option on their own fund performance. As explained by Anson (2001), this call option is granted for free by the investor to the fund manager. It has an exercise price of zero, a maturity of one year (usually) and volatility equal to the volatility of the hedge fund’s before-fee returns.19

• Hedge funds are providing “real options” to non-hedge fund investors by allowing them to readily liquidate their illiquid investments when markets do not fulfill their demand and supply clearing function.

• By trading dynamically, hedge funds are tailoring their return-to-risk profiles to certain classes of investors, and this results in investments that have option-like payoff profiles. In fact, both options and hedge funds are securities with asymmetric non-linear payoffs.

The option-like return pattern of hedge funds obviously presents a challenge for investors. So far, all the models we have been considering (i) were only using factors that intuitively originated from the mix of assets in the funds’ portfolios and (ii) were linear with respect to these factors. By contrast, options are sensitive in a non-linear way to several other risk sources (e.g. volatility, interest rates, convexity of the payoff, etc.). Given what precedes, a large body of the hedge fund literature has suggested adding some contingent claim factors, such as a number of ordinary put or call options into the return-generating process in order to capture the non-linearity with respect to risk factors. Let us now explain how these models work.

8.5.1 The early theoretical models

Henriksson and Merton (1981) were the first to suggest using options to explain the performance of managed portfolios. Their model is basically a simplified two-state framework for performance evaluation in which a fund manager attempts to forecast whether the market return will be higher or lower than the risk-free rate. The manager then adjusts his asset allocation according to his forecast by switching between two discrete levels of systematic risk: an up-market beta when he forecasts an up market and a down-market beta when he forecasts a down market.

19 This perspective actually raises the issue of who is controlling the fund’s volatility. Hedge fund investors should realize that they have provided hedge fund managers with a strong incentive to increase the volatility of the fund, and that this incentive to increase volatility is directly related to the magnitude of the incentive fee.
Henriksson and Merton show that in such a context, the fund’s total return may be viewed as the sum of the return on the market and on a put option on the market. The exercise price of this put option equals the risk-free rate – as the option becomes valuable only if the market return is lower than the risk-free rate. Therefore, the following regression provides consistent estimates for timing and selectivity:

\[ R_i - R_F = \alpha_i + \beta_{i,1} \cdot (R_M - R_F) + \beta_{i,2} \cdot \max(0, -R_M + R_F) + \epsilon_i \]  

(8.18)

Henriksson and Merton show that \( \beta_{i,2} > 0 \) if and only if the fund manager has a superior market timing ability while \( \alpha_i > 0 \) still indicates selection ability. In fact, a positive parameter \( \beta_{i,2} \) can be seen as the number of no-cost put options on the market portfolio provided by the market timing strategy. A negative \( \beta_{i,2} \) and \( \alpha_i \) equal to zero are equivalent to being short a number of put options on the market without receiving any cash.

Henriksson and Merton’s approach has been extended in many directions over recent years. In particular, Glosten and Jagannathan (1994) develop a theoretical framework to analyze the investment style of fund managers by including explicitly the returns on selected option-based strategies as risk factors. The new question is then to determine how many options and which strike prices should be considered.

### 8.5.2 Modeling hedge funds as option portfolios

#### 8.5.2.1 The Fung and Hsieh (1997a) approach

Following the suggestions in Glosten and Jagannathan (1994), Fung and Hsieh (1997a) suggest that the return of hedge funds should be split in three types of components:

- **Some location factors** (“where does the manager trade?”), which tell us the asset classes the manager invests in. These location factors are typically approximated by the returns from a static buy-and-hold policy.
- **Some trading strategy factors** (“how does the manager trade?”), which give us insight on the type of dynamic strategy followed by a manager. These trading factors are typically approximated by the returns from option-based positions.
- **A leverage factor**, which corresponds to a scaling of the two previous factors due to gearing.

To illustrate their claim and identify the location and trading strategy factors, Fung and Hsieh suggest a simple, but innovative, technique that is similar to non-parametric regression. Say we want to analyze the performance of a given hedge fund strategy with respect to a particular asset class. We divide the monthly returns of the asset class into five “states” or “environments” of the world, ranging from severe declines to sharp rallies. Then, the average returns of that asset class, as well as those of the hedge fund strategy, are computed in each state of the world. The final analysis is quite simple:

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20 For instance, Aragon (2002) analyzes the case of fund managers that are timing multiple markets, and Goetzmann et al. (2000), Bollen and Busse (2001) and Ferson and Khang (2001) focus on the biases that may arise when the econometrician observes return data at a frequency different from the frequency at which the manager times the market.

21 Note that Schneeweis et al. (1996) study the ability of CTAs and managed futures to provide downside risk control when combined with an investment in equities. Their conclusion is that CTAs and managed futures may be used as a low-cost alternative to a put option for providing downside protection for an equity investment. However, the level of protection differs significantly. Protection from a put option is certain, but protection from managed futures is not. More recently, Cox and Leland (2000) provide an exhaustive analysis on dynamic investment strategies and their link to options. As an illustration, a fund manager who buys more assets as they rise and sells them progressively as they fall in value would have a payoff that is similar to a long call option. By contrast, a fund manager who short sells more assets as they fall and reduces his short position progressively as they rise in value would have a payoff that is similar to a long put option.
If the hedge fund strategy uses a buy-and-hold strategy in the given asset class, then its return in the five states of the world should align with those in the asset class in a straight line. This corresponds to the above-mentioned location factor.

If the hedge fund strategy uses a dynamic trading strategy in the given asset class, then its return should be non-linearly related to the underlying asset class returns, particularly in the extreme cases (i.e. states 1 and 5).

If the hedge fund strategy is non-related to the given asset class, then its return should be more or less even in the five states of the world.

The shape of the relationship between the returns of a given hedge fund strategy and a particular asset class is easy to represent as a graph. As an illustration, consider Figures 8.13–8.16, which present the most dramatic examples of location and trading strategy factors.

As expected, the dedicated short strategy has no option-like feature. It is negatively linearly related to the S&P 500. The CTA strategy has a return profile similar to a straddle (i.e. long a put and a call) on US equities. The global macro strategy behaves like a short put on the S&P 500 and has an almost linear profile with respect to the USD/JPY exchange rate.

Finally, distressed securities and risk arbitrage also behave like a short put on the S&P 500 (Figures 8.17 and 8.18).

8.5.2.2 The economic rationale

The approach suggested by Fung and Hsieh provides a useful insight on the type of option strategy one should expect when analyzing hedge fund returns. But does it correspond to any sort of economic intuition? Fortunately, the answer is positive. As an illustration, let us consider three strategies, namely, long/short equity, merger arbitrage and systematic traders (CTAs).
Figure 8.14  The performance of the managed futures strategy (in % p.a., right scale) versus US equities (S&P 500, in % p.a., left scale)

Figure 8.15  The performance of the global macro strategy (in % p.a., right scale) versus US equities (in % p.a., left scale)
Figure 8.16  The performance of the global macro strategy (in % p.a., right scale) versus the USD/JPY exchange rate (in % p.a., left scale)

Figure 8.17  The performance of the distressed strategy (in % p.a., right scale) versus US equities (in % p.a., left scale)
Figure 8.18 The performance of the risk arbitrage strategy (in % p.a., right scale) versus US equities (in % p.a., left scale)

Ideally, a long/short equity fund should participate in the upside performance of the market and limit its losses in the downside. The fund will typically increase its market exposure (i.e. beta) when markets are expected to perform well, and reduce its market exposure (i.e. beta) when markets are expected to perform badly. The result is an asymmetric non-linear payoff. Now, consider a portfolio made of one at-the-money call option on a market index. Against the payment of a premium, this call option allows us to participate in the upside potential of the equity index. But if the index drops in value, the maximum loss is limited to the premium initially paid. Clearly, a successful long/short equity manager offers returns that are similar in nature to those of a call option.

Now, consider the case of merger arbitrage. Figure 8.19 shows a scatter plot of merger arbitrage returns versus those of the Russel 3000. A linear OLS regression does not fit the data very well. A local regression performs much better, but it has no economic interpretation. By contrast, an uncovered put option on the Russel 3000 seems to do almost as well as the local regression, and it has an economic interpretation.

Of course, one may wonder what the relationship is between merger arbitrage and selling puts. The answer is quite simple and is extensively developed in Mitchell and Pulvino (2001). Merger arbitrageurs assess the likely outcome of announced mergers and acquisitions, and bet that some merger spreads will converge to zero. In a sense, they are short this spread by going long the target and short the acquirer. When the stock market performs well, mergers and takeovers are successful, and merger spreads converge to zero, which is good for merger

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22 As most long/short equity managers have a long bias, we simply ask for limiting the losses, not for offering a positive performance when markets are bearish. Note that this does not affect the generality of our approach.

23 The merger spread is the difference between the offered price and the market price of securities whose issuers are involved in a merger or a takeover. Because the transaction may still fall through, be delayed or be renegotiated, the market price does not immediately climb to the announced deal price – see Lhabitant (2002c).
Interpreting merger arbitrage (risk arbitrage) as a short put option strategy

arbitrageurs. But, by construction, the merger spread is known and fixed at the initiation of the merger. Thus, the performance of the strategy does not depend on the performance of the market, but is capped at a maximum return. By contrast, when the stock market performs poorly, mergers and takeovers fail and merger spreads widen. This hurts merger arbitrageurs, who face very large potential losses. This clearly shows that merger arbitrageurs behave as if they were selling naked put options on the market. They have a limited upside profit when markets perform well and a large downside risk when markets perform poorly.

Finally, let us consider the case of systematic traders. The majority of them use automated trading systems that attempt to detect trends and profit from them. A typical (simplified) trading rule is to compare the level of a stock index with its moving average. For instance, if the price goes up and crosses the 30-day moving average, this is a buy signal. The long position is closed as soon as the price goes down and crosses the 5-day moving average. A similar rule can be implemented on the short side. That is, if the price goes down and crosses the 30-day moving average, this is a short-sell signal. The short position is closed as soon as the price goes up and crosses the 5-day moving average. At first glance, such a strategy will be profitable as soon as there is a trend and may lose money when the trend changes direction. It is not dependent on any subjective factor.24

At first glance, the payoffs from this type of strategy can easily be approximated by a straddle (i.e. a call plus a put option), as illustrated in Figure 8.13. A straddle is profitable as soon as there is a trend, but loses money if the market does not move much. It loses money if the market does not move much because of the two premiums paid to purchase the options. However, the exercise price of the straddle needs to be constantly rebalanced to take into account the myriad of possible entry and exit decisions of the trend-following strategy (e.g. the systematic exit rules linked to the 5-day moving average in our example).

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24 As an illustration, Jerry Parker from Chesapeake Capital (one of the largest systematic trading advisors in the industry) bought crude oil futures at $20 per barrel in 1990 and saw them rise to $40. When clients asked how he knew to buy crude, Parker replied that he bought oil . . . as it started to move up.
Now, given the prices of a market over any given period of time, the optimal payout of any trend-following system must be one that bought at the lowest price and sold at the highest price. Hence an alternative, suggested by Fung and Hsieh (2000a), is to use a lookback straddle (i.e. a lookback call plus a lookback put). A lookback call is an option that grants the owner the right to purchase an asset at its minimum price over a specified time period. A lookback put is an option that grants the owner the right to sell an asset at its maximum price over a specified time period. The lookback straddle, which is a combination of a lookback call and a lookback put, pays the owner the difference between the minimum and maximum price over a given time period and is identical to the payout. In a sense, it is a continuously rebalanced standard straddle that synthetically replicates the lookback strategy for investors — see Goldman et al. (1979).

Thus, although in practice hedge fund managers and commodity trading advisors may follow a myriad of complex dynamic trading strategies, a few simple option-based strategies capture a large proportion of the variation in their returns over time.25

8.5.2.3 Determining the option portfolio

Agarwal and Naik (2000c) generalize Fung and Hsieh’s (1997a) approach and propose a general asset class factor model that characterizes the systematic risk exposures of hedge funds using both buy-and-hold and option-based strategies.

More specifically, the buy-and-hold risk factors consist of indexes representing:

- Equities, i.e. the Russell 3000 index, the lagged Russell 3000 index, the MSCI World excluding USA index and the MSCI Emerging Markets index.
- Bonds, i.e. the Salomon Brothers Government and Corporate Bond index, the Salomon Brothers World Government Bond index and the Lehman High Yield index.
- The Federal Reserve Bank Competitiveness-Weighted Dollar index and the Goldman Sachs Commodity index.
- Fama–French’s small-minus-big (SMB) and book-to-market (HML) factors, as well as Carhart’s momentum factor (winners-minus-losers).
- The change in the default spread (the difference between the yield on the BAA-rated corporate bonds and the 10-year Treasury bonds).

The option-based strategies originally consisted of an at-the-money option trading strategy (where the present value of the exercise price equals the current index value), an out-of-the-money option trading strategy (where the exercise price is half a standard deviation away from that of the at-the-money option) and a deep-out-of-the-money option trading strategy (where the exercise price is one standard deviation away from that of the at-the-money option) on the Russell 3000 index.26

Given the large number of possible market and trading strategy combinations that hedge funds can follow, Agarwal and Naik use a stepwise regression to ensure a parsimonious selection of factors. They obtain $R^2$ values that are dramatically higher than the ones obtained by Fung and Hsieh (2000b) using Sharpe’s (1992) asset class factor model. Moreover, in the

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25 Note that this also explains why some hedge fund strategies are positively or negatively related to volatility, interest rates, etc. A long option (i.e. convex) payoff is always long volatility, while a short option (i.e. concave) payoff will always be short volatility.

26 In a more recent paper, Agarwal and Naik (2003) implement their model using highly liquid at-the-money (ATM) and out-of-the-money (OTM) European call and put options on the S&P 500 Composite index trading on the Chicago Mercantile Exchange. To avoid collinearity, they do not consider in-the-money (ITM) options as their payoffs can be replicated by a combination of underlying asset and risk-free asset along with an OTM option. For example, the maturity payoff on an ITM call option can be replicated by a long position in the underlying asset, a long position in the risk-free asset and a long position in an OTM put with the same strike price.
case of the hedge funds that follow non-directional strategies, the proportion of observed $R^2$ attributable to trading strategies is, on average, 71% of the total $R^2$. In the case of the hedge funds that follow directional strategies, the average proportion of observed $R^2$ due to trading strategies is 51% of the total $R^2$. Last, but not least, their results show that a large number of equity-oriented hedge fund strategies exhibit payoffs resembling a short position in a put option on the market index, and therefore bear significant left-tail risk, risk that is ignored by the commonly used mean–variance framework. By contrast, managed futures funds are generally "long options". Nonetheless, it is interesting to mention that Agarwal and Naik’s analysis suggests that in a number of styles, excess return remained after adjustment for optionality, at least for the period they observed.

Box 8.5 Replication: the Amin and Kat (2001a) approach

Modeling hedge funds as option portfolios is an interesting intellectual exercise. However, there are three problems with this approach. First, it is not clear how many options (maturities, strike prices, underlying indices) should be included. Ideally, one would want to infer this from the available data, but in practice, the data sample will be too small to avoid ad hoc specification. Second, since only a small number of ordinary puts and calls can be included, there is a definite limit to the range and type of non-linearity that can be captured. Third, how do we value these options? Market quotes may not be arbitrage-free and suffer from a lack of liquidity, while theoretical prices are by definition suspect.

The approach suggested by Amin and Kat takes a completely different perspective – it actually focuses on performance evaluation rather than on explaining hedge fund returns. Instead of considering option portfolios, it concentrates on return distributions and is based on the following reasoning. When investing in a hedge fund, an investor acquires a claim to a certain payoff distribution. Now, can we create a dynamic trading strategy that offers exactly the same payoff distribution? Assuming we live in the world of Black and Scholes (1973), the answer is positive.

Using a continuous time version of the payoff distribution pricing model introduced by Dybvig (1988a,b), Amin and Kat determine the cost of the cheapest dynamic trading strategy, trading some reference index and cash, which generates the same payoff distribution as the hedge fund in question. They analyze the return distributions of 77 hedge funds and funds of funds in operation between May 1990 and April 2000 listed in the Zurich Capital Markets hedge fund database. Their conclusion is striking: investors could get the same risk-adjusted performance, on average 6.42%, more cheaply by using a dynamic trading strategy rather than by actually investing in the hedge funds in 72 out of the 77 cases . . .

8.6 DO HEDGE FUNDS REALLY PRODUCE ALPHA?

After having investigated all these asset pricing models, the question of whether hedge funds effectively produce alpha is open.

Let us recall that in practice, there are two ways of achieving superior returns in financial markets:

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27 This should be kept in mind when analyzing the impact of adding options in traditional portfolios. By definition, a short put option carries the risk of rare but large losses.
The first approach consists of increasing beta, that is, accepting to take a particular risk for which the market compensates you. For instance, equities have a higher expected return than cash over time for the simple reason that they are a more risky investment than cash and that part of this risk is non-diversifiable (systematic). The same is true of long duration bonds versus cash, corporate bonds versus treasuries, etc.

The second approach consists of generating alpha, i.e. outsmarting other market participants. Market timing, arbitrage and active security selection are examples of alpha strategies.

As we have seen, when we strip a hedge fund or a strategy and identify all the systematic risk it is exposed to, we often find that the fund manager is not doing anything particularly unique beyond taking in particular risk premiums. These risk premiums cover beta exposures to liquidity risk, credit risk, volatility risk, currency risk, interest rate risk, etc. Thus, most hedge funds have a lot of betas embedded in their returns, and these beta sources are relatively easy to access and replicate.

When a hedge fund’s alpha is assessed using only the stock market as a risk factor, all these risk premiums appear as being alpha, because they are not correlated to stock market risk. But in fact, in a correctly specified model, this apparent alpha would largely be categorized as beta. Thus, it seems that the superiority of hedge funds therefore essentially relies on packaging up beta . . . and selling it at alpha prices.

As an illustration of this lack of uniqueness, hedge funds in a given strategy are, for the most part, extremely correlated to one another.

As an illustration, Fung and Hsieh (2002b) observe that fixed income hedge funds primarily have exposure to fixed income-related spreads, including the convertible bond/Treasury spread, the high yield/Treasury spread, the mortgage/Treasury spread and the emerging market bond/Treasury spread.